O K L A H O M A   S T A T E   U N I V E R S I T Y

SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

ECEN 4503
Random Signals and Noise
Spring 2002

Final Exam

Choose any four out of five problems,
Please specify
1)_____; 2)_____; 3)_____; 4)_____;

Name : ______________________________

Student ID: ________________________________

E-Mail Address:_______________________________________

Problem 1:
In a computer simulation it is desired to transform numbers, that are values of a random variable *X* uniformly distributed on $(0,1)$, to numbers that are values of a Cauchy random variable *Y* as defined by

$$F_Y(y) = \frac{1}{2} + \frac{1}{\pi}\tan^{-1}(\frac{y}{b}).$$

Find the required transformation *T*.

Problem 2:

The random variables X and Y are statistically independent with exponential densities

$f_X(x) = \alpha e^{-\alpha x} u(x)$, and

$f_Y(y) = \beta e^{-\beta y} u(y)$.

Find the probability density function of the random variable $Z = \min(X, Y)$.

Problem 3:

The random variables *X* and *Y* are statistically independent with Rayleigh densities

$$f_X(x) = \frac{x}{\alpha^2} e^{-x^2/2\alpha^2} u(x) \text{, and}$$

$$f_Y(y) = \frac{y}{\beta^2} e^{-y^2/2\beta^2} u(y) \text{.}$$

Show that if $Z = X/Y$, then

$$f_Z(z) = \frac{2\alpha^2}{\beta^2} \frac{z}{(z^2 + \alpha^2/\beta^2)^2} u(z) \text{.}$$

Problem 4:

A random process is defined by

$$Y(t) = X(t)\cos(\omega_0 t + \Theta),$$

where $X(t)$ is a wide-sense stationary ranom process that amplitude-modulates a carrier of constant angular frequency ω_0 with a random phase Θ independent of $X(t)$ and uniformly distributed on $(-\pi, \pi)$. Find $E[Y(t)]$ and autocorrelation function of $Y(t)$. Is $Y(t)$ wide-sense stationary?

Problem 5:

A random process is given by

$$X(t) = A\cos(\Omega t + \Theta)$$

where A is a real constant, Ω is a random variable with density function $f_\Omega(\cdot)$, and Θ is a random variable uniformly distributed on the interval $(0, 2\pi)$ independently of Ω. Show that the power spectrum of $X(t)$ is

$$S_{XX}(\omega) = \frac{\pi A^2}{2}[f_\Omega(\omega) + f_\Omega(-\omega)].$$